DYNAMIC ANALYSIS OF ELASTOPLASTIC SYSTEMS SUBJECTED TO EARTHQUAKE GROUND MOTION FOR A 20-STORY STEEL BUILDING

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Abstract- The paper presents a numerical method for a non-linear system equipped with passive viscous fluid dampers (VFD) in attempts to improve seismic resistance. A mathematical model for the motion and a computational algorithm to find responses of the structure are proposed. Focusing on a steel structure which is a benchmark 20-story building designed for the SAC project for the Los Angeles, California, the numerical example analyzes the dynamic response of the building subjected to earthquake-induced ground motion and simultaneously passively controlled with VFD. Furthermore, the paper will provide a basis for evaluating the factors affecting responses of such a controlled system and therefore these responses are compared to an elastic structure without VFD, an inelastic structure without VFD, an elastic structure with VFD, and an inelastic structure with VFD.
Keywords – Dynamics of Structures, Structural Control, Elastoplastic System, Steel Structures, Viscous Fluid Dampers

1. INTRODUCTION

Results of global climate change is the advent of the more and more earthquakes, causing damage to buildings. Structural control is introduced as well effective solution to improve seismic resistance for buildings. Structural control is categorized under three basic headings: passive control, active control, and semi-active control. This paper uses a passive control system, a reliable control system, because it does not need input energy for its dampers. To evaluate dampers' performance for both strong and weak earthquakes, the reliable mathematical model and algorithm to find the response of non-linear systems using fluid viscous dampers are required. The research about passive VFD and a non-linear structure is only the use of a software to find response [3][4][5], not having a reliable numerical method for the problem. Consequently, this paper investigates a numerical method for a bilinear system equipped fluid viscous dampers, where a bilinear behavior of a system plays a role as a damper dissipating earthquake input energy thanks to plastic straining. Formulations are introduced for expressing the inelastic behavior of structures. Furthermore, computer programing would help to change controlling parameter in VFD dampers or section properties more rapidly than using a structural software. Eventually, there would be less costly for purchase of a software license.

Consider a column element of Figure 1 with height H, elastic modulus E, and second moment area for the column Ic. For this element, the bending moments M1, M2 and shears V1, V2 at two ends are related to two lateral displacement x1, x2 as follows

$$\left.\begin{aligned} V_2 &= \frac{12EI_c}{H^3}\left(x_2 - x_1\right) = -V_1 \\ M_2 &= \frac{6EI_c}{H^2}\left(x_2 - x_1\right) = M_1 \end{aligned}\right\} \Rightarrow V_2 = \frac{2M_2}{H}$$
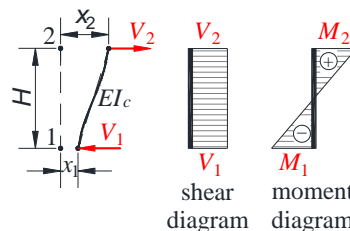

Figure 1: the internal forces in a column associated with its ends' lateral displacements

When yield occurs $M_2 = M_p = \sigma_y W_p$, the linearly elastic limit is given by $f_y = \dfrac{2\sigma_y W_p}{H}$ (1)

where σy is yield strength of a column's material; and Wp is plastic modulus of a column's section.

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For a inelastic system, the relationship between laterally resisting force fs and resulting displacement x is displayed as Figure 2, in which the initial stiffness ki can be calculated with the following expression [2]

$$k_i = \frac{12EI_c}{H^3}$$

(2)

ky is the yield stiffness; and xlimit is yield deformation and equal to fy/ki.

The model 1 is the model 2 when ky=0. Therefore, in general the model 2 is used to present the lateral force-displacement relation in this research.

For the bilinear model (Figure 2b), the mathematical expression for inelastic structures is in form as

$$f_s(t+\Delta t) = f_s(t) + k\left[x(t+\Delta t) - x(t)\right]$$

(3)

$$f_y^{up}(t+\Delta t) = f_y + k_y\left[x(t+\Delta t) - x_{\text{limit}}\right] \text{ is uper elastic limit}$$

$$f_y^{lo}(t+\Delta t) = -f_y + k_y\left[x(t+\Delta t) + x_{\text{limit}}\right] \text{ is lower elastic limit}$$

$$\text{where } \begin{cases} k = k_y & \text{for } \begin{bmatrix} f_s \geq f_y^{up} \\ f_s \leq f_y^{lo} \end{bmatrix} \\ k = k_i & \text{for } \begin{cases} f_s < f_y^{up} \\ f_s > f_y^{lo} \end{cases} \end{cases}$$
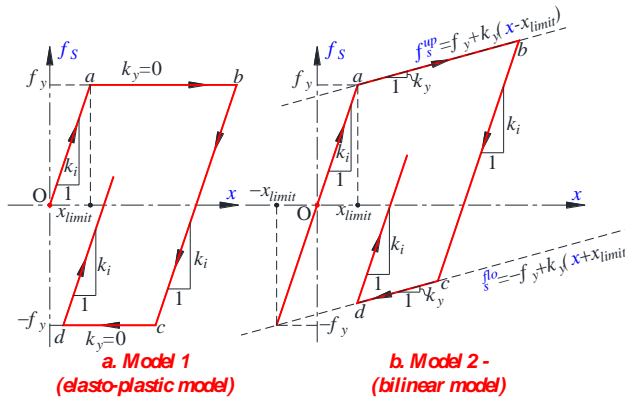

Figure 2: Two approximate model of force-deformation relation

For passive control fluid viscous dampers, the damping force is proportional to its ends' velocity as [3]

$$F_{VFD} = C_{VFD}|\dot{x}|^{\alpha}\,\text{sign}(\dot{x})$$

(4)

where $\dot{x}$ is end to end velocity across the element; CVFD is the damping coefficient; α is powers in the range of 0.3 to 2.0; and $\text{sign}(\dot{x})$ is the function equal to 1 if $\dot{x}$ is positive, equal to -1 if $\dot{x}$ is negative, and equal to 0 if $\dot{x}$ is zero.

On purpose of assessing the effectiveness of response reduction, requirements of a reliable model and algorithm of a passive VFD structure is intended to establish the differential equation of motion and to solve this equation. The goal of numerical examples means to estimate the effectiveness in reducing dynamic responses of the benchmark 20-story structure equipped with passive VFD. This result is compared to such a structure with no control and with various schemes of passive VFD in elastics and in plastics.

2. INELASTIC STRUCTURES WITH PASSIVE FLUID VISCOUS DAMPERS
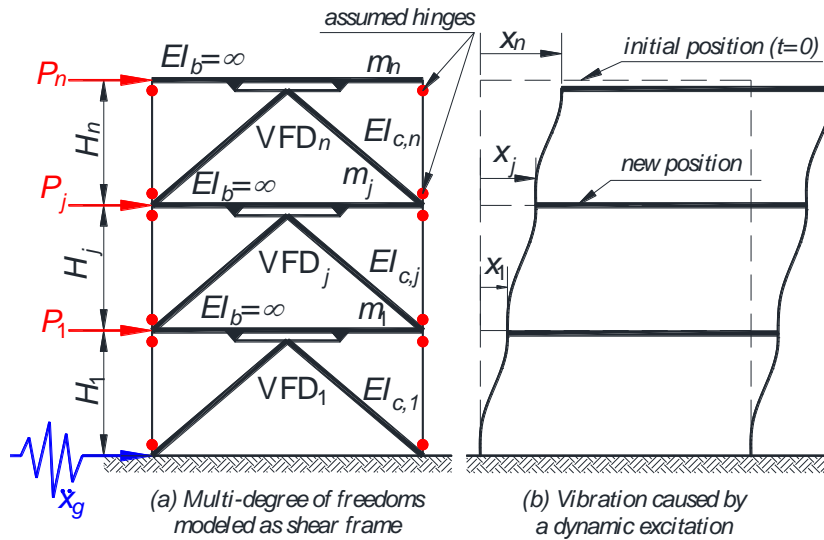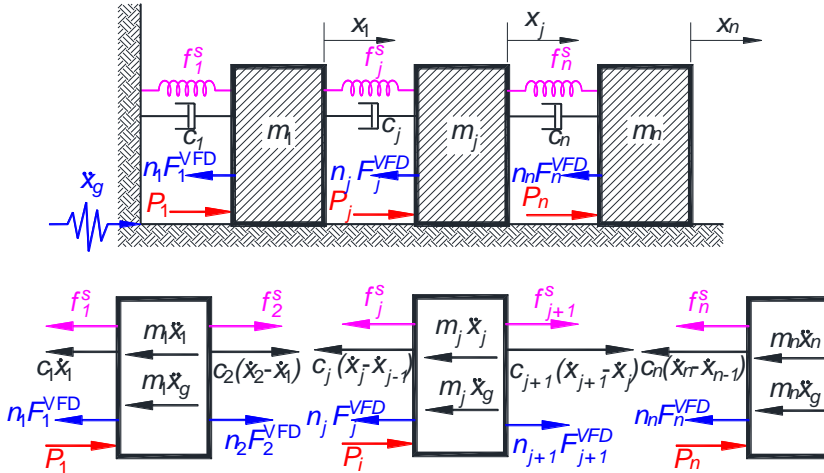*2.1 The governing differential equation of motion*

Figure 3

Figure 4: Mathematical Model

Consider the multi-story frame working as shear building model. This structure is modeled as multi-degree of freedom in which the masses are concentrated at each of the floor level. From the assumption, an n-story and multi-bay shear building is simplified to an n-story and single-bay building shown in Figure 3, where a lateral initial stiffness at the jth floor $k_{i,j}$ is equal to the total of the stiffnesses of individual columns

$$k_{i,j} = \sum_{columns} \frac{12EI_{c,j}}{H_j^3}(j=1,...,n)$$

(5)

The structure is seismically retrofitted using the number of VFD n_j at the jth floor. For any floor without a damper, n_j is assigned zero. From the mathematical model (Figure 4) and using Dalambert's principle, the differential equation governing the lateral motion of the structure equipped with passive fluid viscous dampers is given in matrix form

$$\mathbf{M}.\ddot{\mathbf{x}} + \mathbf{C}.\dot{\mathbf{x}} = -\mathbf{M}.\mathbf{l}.\ddot{x}_g + \mathbf{P} - \mathbf{F_S} - \mathbf{F_{VFD}}$$

(6)

where $\mathbf{M} = diag[m_1,...,m_n]$ is the constant lump mass matrix, where mi is the mass at the jth floor;

$$\mathbf{C} = \begin{bmatrix} c_1 + c_2 & -c_2 & ... \\ ... & c_j + c_{j+1} & ... \\ ... & -c_n & c_n \end{bmatrix}$$

is the constant damping matrix and is calculated with the help of Rayleigh's method;

$\mathbf{l} = [1...1...1]^T$; an overdot denotes differentiation with respect to time; thus $\ddot{x}_g$ is earthquake ground acceleration;

$\mathbf{x} = \{x_1,...,x_j,...,x_n\}^T$, $\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt}$, and $\ddot{\mathbf{x}} = \frac{d^2\mathbf{x}}{dt^2}$ are structure's displacement, velocity, and acceleration vectors, respectively;

$\mathbf{P} = \left\{ P_1...P_j...P_n \right\}^T$ is an externally applied force vector, where Pi is an external force at the jth floor; for elastic model,

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & ... \\ ... & k_j + k_{j+1} & ... \\ ... & -k_n & k_n \end{bmatrix}$$ and for bilinear model $\mathbf{K.x}$ is replaced by a resisting force vector FS as

$\mathbf{F_S} = \left[f_1^s - f_2^s,...,f_j^s - f_{j+1}^s,...,f_n^s \right]^T$ (7), where f_j^s is determined with (3); and the damping force vector FVFD generated by

$$\mathbf{F_{VFD}} = \left\{ \begin{array}{c} n_1 F_1^{VFD} - n_2 F_2^{VFD} \\ ... \\ n_j F_j^{VFD} - n_{j+1} F_{j+1}^{VFD} \\ ... \\ n_n F_n^{VFD} \end{array} \right\}$$

fluid viscous dampers is given by [3] (8), where F_j^{VFD} is given by (4).

2.2 A computational algorithm for inelastic structures with passive fluid viscous dampers

With computer implementation and a program language, dynamic response analysis of non-linear systems is performed with a numerical solution, and consequently time instants are divided into discrete values so that the time interval ⬜ t is constant at all time instants. As a result of both a non-linear structure (FS not constant) and a non-linear damper ($\alpha j \neq 1$), in order to find the responses of the structure the paper presents a time-stepping method based partially on NewMark's method with assumption of linear acceleration for the time interval ⬜ t. Subsequently the numerical method is NewMark's modified method in attempt to match it to the non-linear systems. A computational error is significantly reduced with the coverage of energy responses as the time step ⬜ t sufficiently decreases[1]. The calculation is approved via the following flowcharts (Figure 5).
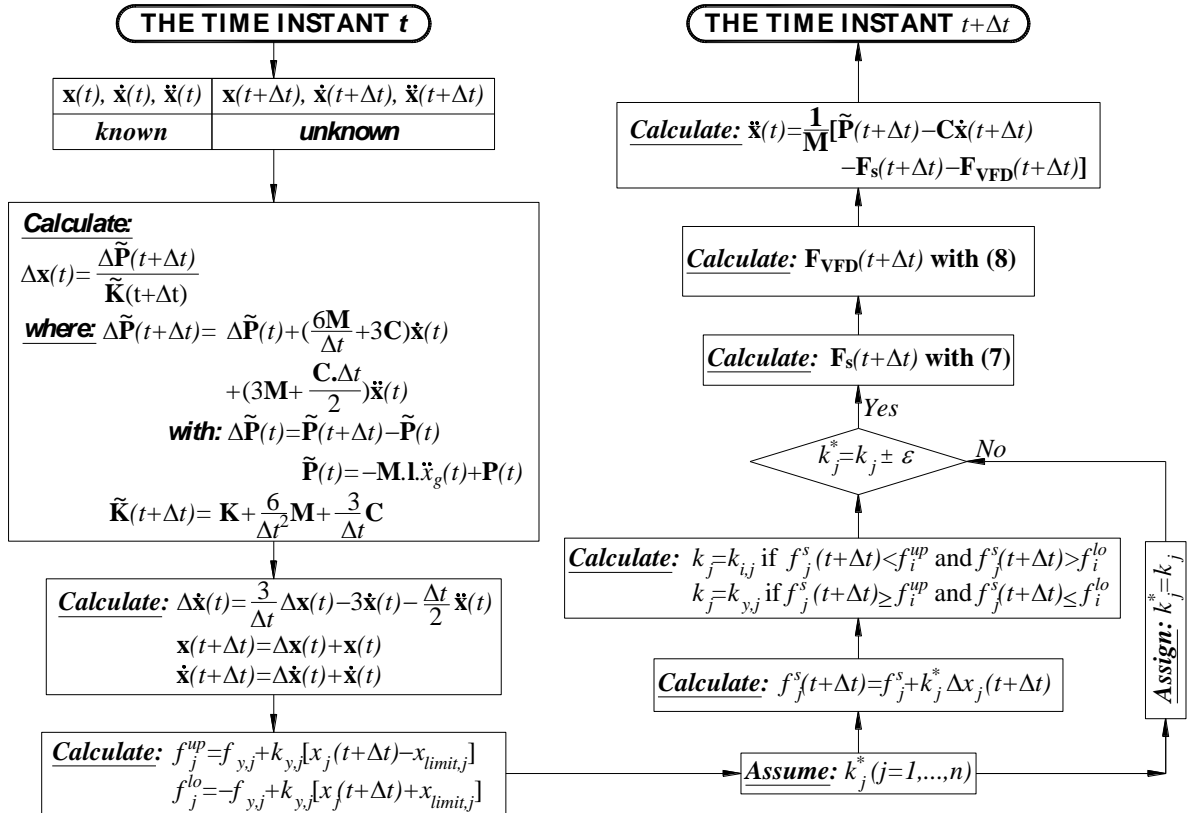
Figure 5: Flowchart for the numerical method

3. NUMERICAL EXAMPLES

The 20-story structure used for this benchmark study were designed by Brandow and Johnston Associates 1996 for the SAC Phase II Steel Project [6]. These buildings were chosen because they also serve as benchmark structures for the SAC studies and, thus, will provide a wider basis for the comparison of results. All simulations were performed by using routines written in MATLAB.

3.1 Description of the benchmark 20-story structure

The benchmark 20-story structure is made of steel having $E = 200GPa$, $\sigma_y = 345MPa$, and $k_{y,j} = 10\% k_{i,j}$ $\left(j = \overline{1,20}\right)$. The damping ratios of the steel frame for mode 1 and 2 are $\zeta 1 = \zeta 2 = 5\%$. The dynamic properties of structure are given in Table 1.

The weight at the jth floor is $W_j = m_j g$ with the gravity acceleration $g = 9.81 {}^m\!/\!_{s^2}$.

Table 1. The dynamic properties of the 20-story structure

Floor	Section	$m_j \left(\times 10^3 kg\right)$	$k_{i,j} \left({kN}\!/\!{cm}\right)$	$f_{y,j} \left(\times 10^3 kN\right)$	floor level $Z_j (m)$
1st	W24x335	563	30 173	88.23	5.49
2nd	W24x335	552	80 400	122.32	9.45
3rd	W24x335	552	80 400	122.32	13.41
4th	W24x335	552	80 400	122.32	17.37
5th	W24x229	552	51 686	81.07	21.33
6th	W24x229	552	51 686	81.07	25.29
7th	W24x229	552	51 686	81.07	29.25
8th	W24x229	552	51 686	81.07	33.21
9th	W24x229	552	51 686	81.07	37.17
10th	W24x229	552	51 686	81.07	41.13
11th	W24x192	552	42 295	67.03	45.09
12th	W24x192	552	42 295	67.03	49.05
13th	W24x192	552	42 295	67.03	53.01
14th	W24x131	552	27 160	44.37	56.97
15th	W24x131	552	27 160	44.37	60.93
16th	W24x131	552	27 160	44.37	64.89
17th	W24x117	552	23 917	39.22	68.85
18th	W24x117	552	23 917	39.22	72.81
19th	W24x84	552	16 012	26.87	76.77
20th	W24x84	584	16 012	26.87	80.73

The first three natural frequencies are $\omega_1 = 7.1 {}^{rad}\!/\!_s$; $\omega_2 = 18.2 {}^{rad}\!/\!_s$; $\omega_3 = 30.3 {}^{rad}\!/\!_s$ and correspondingly the first three natural period of the uncontrolled structure are $T_1 = 0.88s$; $T_2 = 0.34s$; $T_3 = 0.21s$. Responses of the structure are computed with 4 cases, consisting of (A) elastic behavior and without VFD; (B) plastic behavior and without VFD; (C) elastic behavior and with VFD; and (D) plastic behavior and with VFD.

The viscous dampers coefficients of passive control VFD in these cases (C) and (D) are set up to increase structural damping levels to 20% of critical, i.e. $C_j^{VFD} = 4c_j$ and $\alpha j = 0.9$ $\left(j = \overline{1,20}\right)$. The responses of elastic behaviors (A) and (C) are investigated with fy,j=∞; and the responses of the structure without VFD (A) and (B) are investigated with $C_j^{VFD} = 0$. The responses of the 20-story structure are analyzed with Kobe earthquake excitation (Figure 6) and the zero initial conditions. To cut down the error of this numerical method as the state changes from ki to ky or conversely, the energy response must converges, and for this to work the time interval □ t is taken to be 0.00125s. The total time for dynamic analysis is 28 cycles of the first natural period T1.
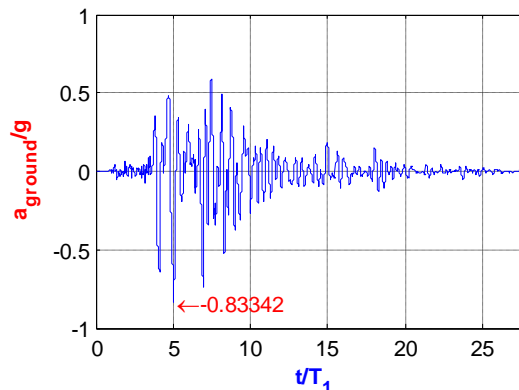

Figure 6: Ground acceleration of Kobe earthquake

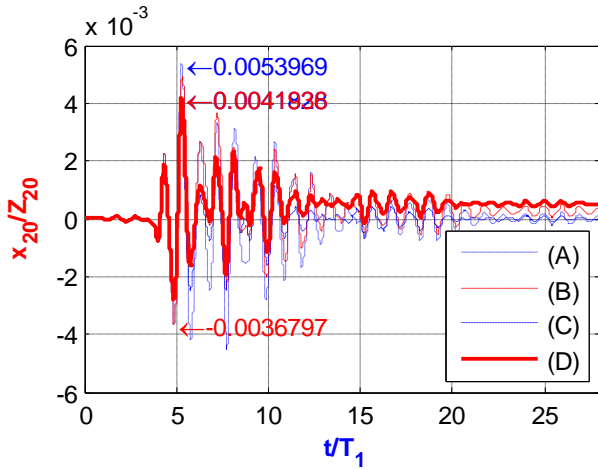*3.2 Responses of the 20-story steel structure with Kobe*

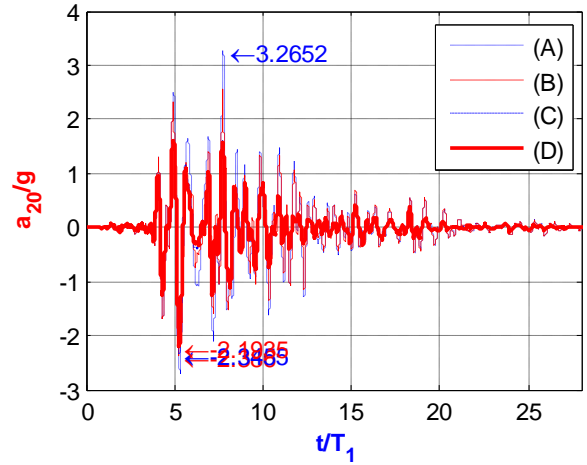Figure 7: The top drift relative to the top floor level
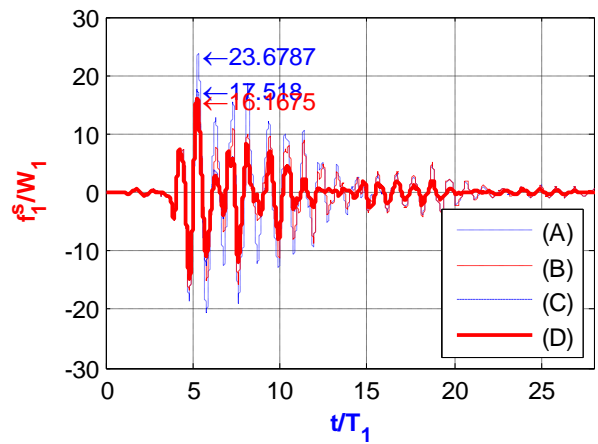
Figure 8: The top acceleration response

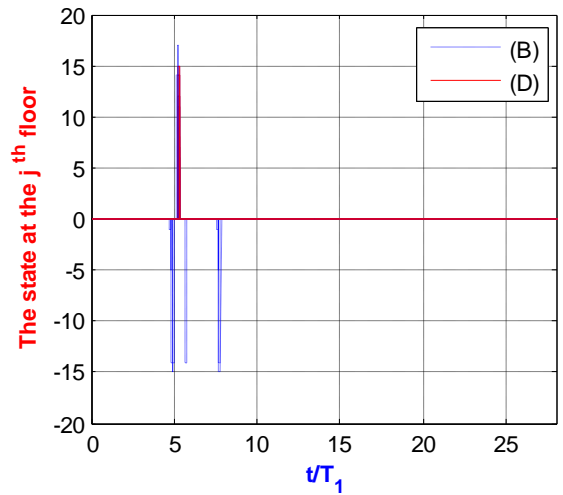Figure 9: The 1st floor shear force response

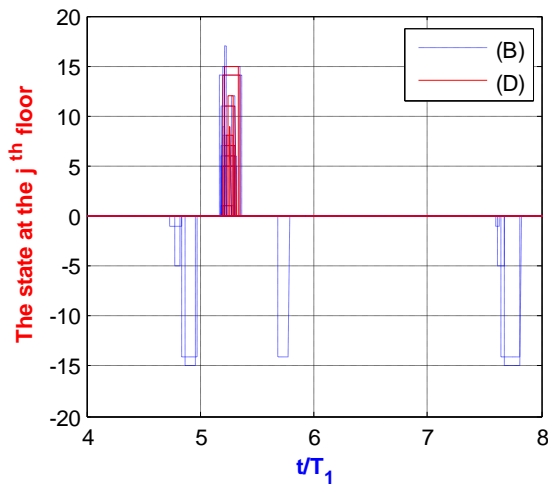Figure 10: The state of the structure during Kobe earthquake

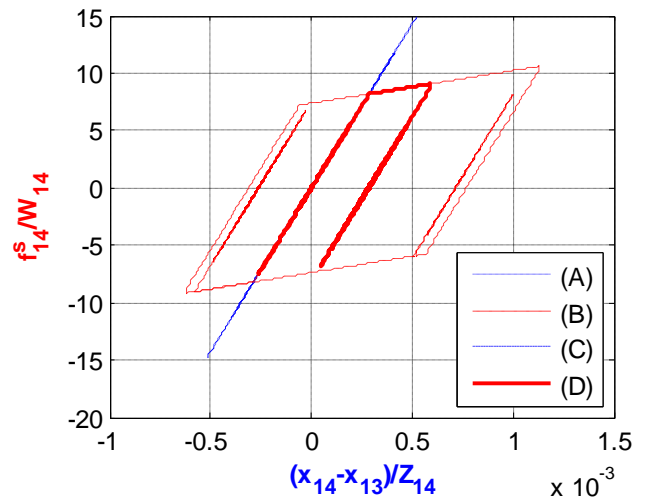Figure 11: The state of the structure from the 4th to 8th cycle

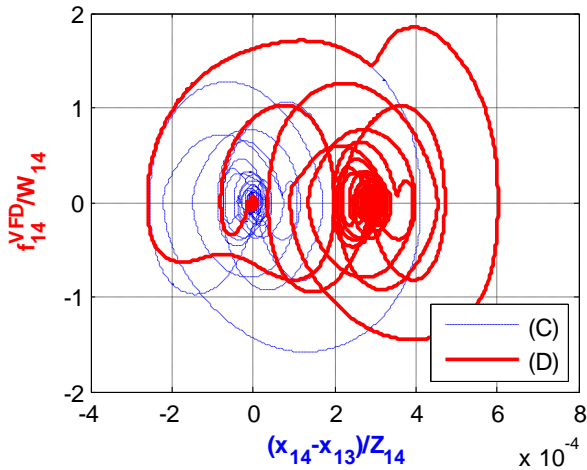Figure 12: The hysteresis loop of strain at the 14th floor

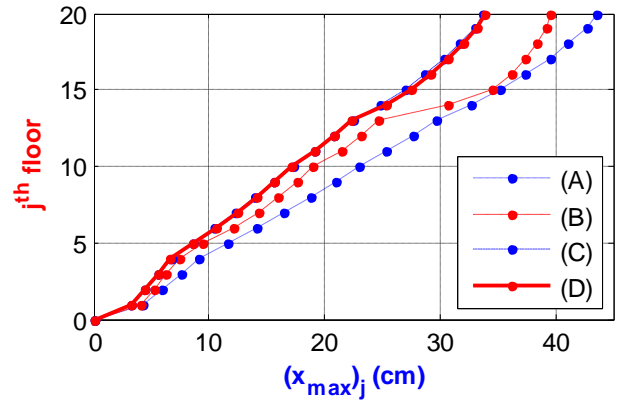Figure 13: The hysteresis loop of VFD at the 14th floor

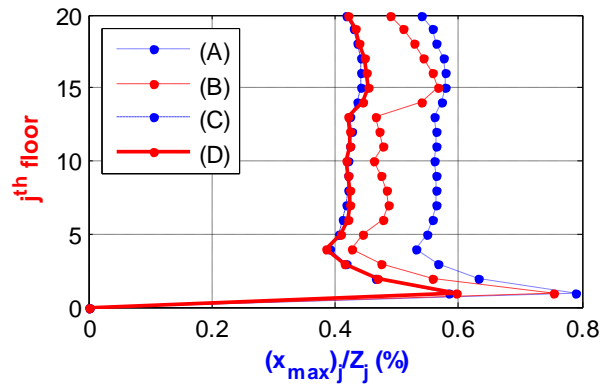Figure 14: The drift of the 20-story structure

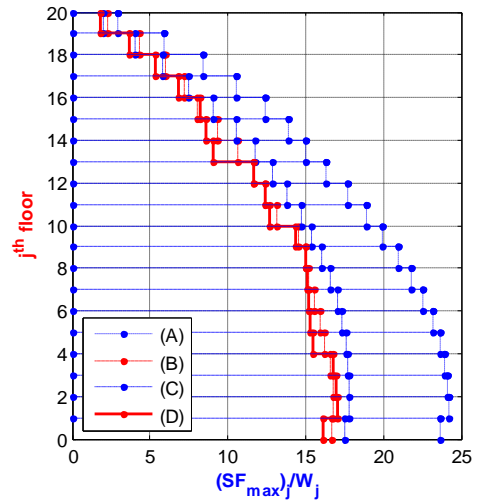Figure 15: The drift relative to the floor level

Figure 16: The shear force relative to the weight

First, for the top drift response, after the earthquake excitation, the non-linear system with bilinear model shows that the building has the plastic strain and is disturbed from its new equilibrium position for both case (B) as 2.3cm and case (D) as 4.6cm (Figure 7). The top drift is accumulation of lateral displacements of the 19 floor below. Also, the displacement response reduction of case (D) is higher than that of case (C) because the bilinear model functions as a damper absorbing input energy. Moreover, the 1st drift relative to its height is the greatest (Figure 15) owing to its smallest lateral stiffness. As a result of this, during motion, the above floor from 2nd to 20th is one block, and consequently the 1st shear force is the greatest (Figure 15), causing the structure to collapse rapidly. In order the structure to significantly diminish in shear force, any one of the solutions would be {1} a classical solution of increasing the 1st lateral stiffness; or {2} a structural control solution of increasing the number of VFD n1 in attempts to upgrade horizontal displacement resistance for the structure.
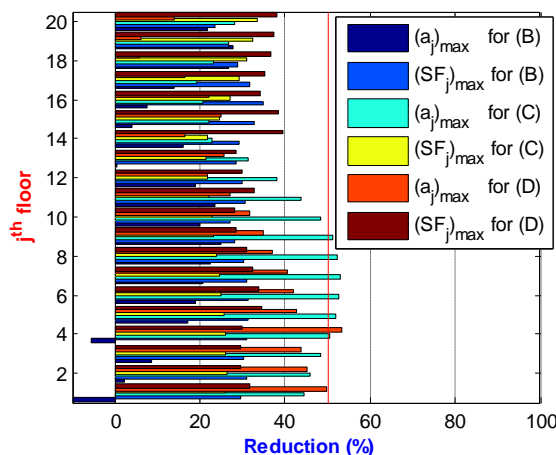
Figure 17: The maximum response reduction of (B), (C), and (D) relative to (A)

Second, for the state with value equal to zero as elastic or different from zero as plastic, the state of one means that the 1st floor is plastic for instance (Figure 10 and Figure 11). The structure undergoes a great number of plastic state from 4 to 8 of the cycle when the value of ground acceleration is large, and from these plastic state at 14th floor occurs most frequently. Thus, the bilinear model material is analyzed at this floor as shown in Figure 12. According to the Figure 12, the spring forces fs in plastic analysis (B) and (D) is smaller than these in elastic analysis (A) and (C). The area of hysteresis loop of case (D) is smaller than that of case (B). It is obvious that VFD partially dissipates the input seismic energy and therefore the plastic strain energy is significantly reduced. Third, for the top floor acceleration response (Figure 8), plastic straining also causes acceleration reduction to the structure, acceleration response in case (B) compared to one in case (A) because the structure turns out less stiffer. Finally, although in case (C) the maximum acceleration responses at these 1st and 4th floor increase owing to the powers α_j of VFD, the maximum response reductions of cases (B), (C), and (D) compared to case (A) are reliable (Figure 17).

4. CONCLUSION

Although the dynamic analysis of a nonlinear system creates elaborated calculation compared with that of a linear system, the calculation is necessary for assessment of response reduction of a structure using VFD for intense earthquakes. This paper provides a way of programing on purpose of conducting research as to dynamic analysis rather than a way of buying commercial software license, especially for developing countries. Also, this approach is restricted to shear building approach and to VFD used, but it could be developed further by changing the stiffness matrix in general approach or changing the damping force vector of VFD to another damper. In addition, when a practice building is subjected to a seismic load, the building deforms and equilibriums at a new position if it does not collapse, and obviously that is illustrated in the numerical example by plastic strain, a residual deformation. Additionally, the response of a structure retrofitted with VFD depends not only on controlled parameters of VFD, and the lateral stiffness, but also on the linearly elastic limit. Generally, for the 20-story, VFD not only lowers plastic deformation in columns and the drift, but also lessens the x_{drift}-to-z_{drft} ratio at the floor having small horizontal stiffness. Hence, VFD is suitable to enhance seismic resistance for retrofitted structures.

5. REFERENCES

[1] Le Minh Thanh, Pham Nhan Hoa, Nguyen Dinh Hung, Le Huu Huy, Chu Quoc Thang, "A numerical method for inelastic structures subjected to earthquake excitation", Vietnam Computational National Conference IX, Vol.1, Dynamics of Structures – Ha Noi, Vietnam, Dec. 2012.

[2] Anil K.Chopra, "Dynamics of Structures", 4th edition, Prentice Hall Press, 2012.

[3] Robert J. Mcnamara and Douglas P. Taylor, "Fluid viscous dampers for high-rise buildings", the structural design of tall and special buildings, 12, 145–154, 2003.

[4] M.C. Constantinou, P. Tsopelas, and W. Hammel, "Testing and modeling of an improved damper configuration for stiff structural systems", p.76-97, 1997, http://taylordevices.com/literature.html

[5] H. Kit Miyamoto and Douglas P. Taylor, "Structural control of dynamic blast loading using fluid viscous dampers", http://taylordevices.com/literature.html

[6] Y.Ohtori, R. E. Christenson, B. F. Spencer, "Benchmark Control Problems for Seismically Excited Nonlinear Buildings", Journal of Engineering Mechanics © ASCE / April 2004.